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Consequence of the Wigner Rotation: Perturbative QCD Analysis of the Pion Form Factor

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Abstract We analyse the perturbative contributions from the higher helicity ($\lambda_1 + \lambda_1 = \pm 1$) components, which should be naturally contained in the light-cone wave function for the pion as a consequence of the Wigner rotation, in the QCD calculation of the pion form factor. It is pointed out that the contributions may provide the other fraction needed to fit the pion form factor data besides the perturbative contributions from the ordinary helicity components evaluated using the factorization formula with the asymptotic form of the distribution amplitude. We suggest a way to test the higher helicity state contributions.

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I. INTRODUCTION

The application of perturbative quantum chromodynamics (pQCD) to exclusive processes at larger momentum transfer has developed for more than a decade, and there has been significant progress in this field [1]. The QCD analyses based on the asymptotic behavior of QCD and the factorization theorem [1-4] have been successful in reproducing a number of important phenomenological features such as the dimensional counting rules [5] by separating the nonperturbative physics of the hadron bound states from the hard scattering amplitude which controls the scattering of the underlying quarks and gluons from the initial and final directions. However, the applicability of the pQCD framework to exclusive processes, such as the pion form factor, is still under debate [1,6-11] as a consequence of unsatisfied quantitative calculations arising from the ambiguous understanding of the nonperturbative hadronic bound states.

The nonperturbative part of the QCD theory is contained in the process-independent "distribution amplitudes" which include all of the bound state nonperturbative dynamics of each of the interacting hadrons. The distribution amplitude for the pion has been studied since the beginning of the application of pQCD to exclusive processes. The earliest asymptotic pion distribution amplitude [3] was found insufficient to reproduce the magnitude of the existing pion form factor data [6]. The Chernyak-Zhitnitsky (CZ) distribution amplitude [12, 13], which is constructed by fitting the first three moments using the QCD sum rule technique, is good in reproducing the correct magnitude as well as the scaling behavior of the pion form factor, and thus it has received attention. However, it was found by Isgur and Llewellyn Smith [8] that the perturbative contribution still seems unlikely to dominate at available momentum transfers by excluding the contributions of the end-point regions where sub-leading (higher twist) terms are a priori likely to be greater than the perturbative result. Huang and Shen [9] examined this objection to the applicability of pQCD by using a CZ-like distribution amplitude with a sufficient suppression factor in the end-point regions and found that the perturbative contribution seems to dominate at Q^2 of a few $(\mathrm{GeV/c})^2$. Szczepaniak and Mankiewicz [10] also attempted, similarly, to cure possible end-point irregularities by introducing a cut distribution amplitude which vanishes in the cutoff end-point regions and found that the dependence on the end-point cutoff is significantly reduced for the calculated pion form factor. Li and Sterman [11] attempted to explain the suppression in the end-point regions by including the effects of the Sudakov form factor of the quarks. Their results show that the applicability of pQCD at

currently available momentum transfers is closely related to the end-point behavior of the hadronic wave function.

However, whether the CZ-like distribution amplitude is the correct pion distribution amplitude is still an open problem and its correctness should not be judged solely by its success in reproducing the correct magnitude of the pion form factor. There have been arguments against the CZ distribution amplitude, coming from more sophisticated analyses in the QCD sum rule framework[14]. Some earlier lattice Monte Carlo calculations [15], designed to compute the pion distribution amplitude directly, were unable to distinguish between the asymptotic form and the CZ form. In a recent improved lattice QCD calculation [16], the second moment of the pion distribution amplitude was found to be smaller than previous lattice QCD calculations [15] and sum-rule calculations [12, 13], and this suggests that the pion distribution amplitude may close to the asymptotic form rather than to the CZ form. This result seems also to be supported by a recent revised light-cone quark model [17] evaluation in which the the pion distribution amplitude is found to be close to the asymptotic form with several physical constraints on the pion wave functions also satisfied. From another point of view, it has been claimed in [18] that it is impossible to obtain the large value 0.4 of the second moment of the CZ distribution amplitude without getting a very large soft contribution to the pion form factor in the region $Q^2 \geq 2$ (GeV/c)^2. Therefore it is necessary to study the problem of the applicability of pQCD further.

It has been speculated [17] that the perturbative contributions from the higher helicity ($\lambda_1 + \lambda_2 = \pm 1$) components, which were found [17, 19] should be naturally contained in the full light-cone wave function for the pion as a consequence of the Wigner rotation [20], may provide the other fraction needed to fit the pion form factor data besides the perturbative contributions from the ordinary helicity components evaluated using the asymptotic form distribution amplitude. This speculation is different from the previous consideration that the perturbative contributions from the higher helicity components should be negligible by using the conventional factorization formula directly, since the distribution amplitudes calculated from the higher helicity component wave functions vanish. Recently, it has been observed by Jakob and Kroll [21] that the intrinsic transverse momentum leads to a substantial suppression of the perturbative QCD contribution. This implies that we still need some other contribution in addition to the perturbative QCD calculation, even in the case of a CZ-like distribution amplitude. The purpose of this paper is to derive the contribution from the higher helicity components and suggest a way to test this

contribution.

This paper is organized as follows. In Sec. II we briefly review the pion wave function in a revised light-cone quark model with emphasis on the spin structure of the pion in light-cone formalism. In Sec. III we analyse the contribution from the higher helicity components. In Sev. IV we perform a numerical calculation and discuss the possibility to test the higher helicity state contributions. Sec. V is reserved for summary and discussion.

II. THE PION WAVE FUNCTION IN LIGHT-CONE FORMALISM

The light-cone formalism [1-3] provides a convenient framework for the relativistic description of hadrons in terms of quark and gluon degrees of freedom, and the application of pQCD to exclusive processes has mainly been developed in this formalism. It has been shown [17, 19] that the spin structure of the pion in light-cone formalism is quite different from that in the SU(6) naive quark model by taking into account the effect of the Melosh rotation [22] relating spin states in light-cone (light-front) formalism and those in equal-time (instant-form) formalism. A natural consequence is the presence of the higher helicity ($\lambda_1 + \lambda_2 = \pm 1$) components in the valence state light-cone wave function for the pion besides the ordinary helicity ($\lambda_1 + \lambda_2 = 0$) components. It has been shown in Ref.[19] that the some low energy properties of the pion, such as the electromagnetic form factor, the charged mean square radius, and the weak decay constant, could be well interrelated within a light-cone constituent quark model, taking into account the higher helicity contributions. The same effect has been also applied [23] to explain the proton "spin puzzle" arising from the Ellis-Jaffe sum rule violation found by the European Muon Collaboration.

In this paper we analyse the perturbative contributions from the higher helicity states to the pion form factor at large Q^2, where the high momentum tail of the wave function is of relevant and the results of the light-cone constituent quark model in Ref.[19] are no longer adequate. We first briefly review the light-cone wave function for the pion for the purpose of our latter analysis. It has been argued [17, 19] that the the lowest valence state in the light-cone wave function can be expressed as

$$\begin{aligned} |\psi^{\pi}_{q\overline{q}}>= {}& \psi(x,\vec{k}_{\perp},\uparrow,\downarrow)|\uparrow\downarrow> + \psi(x,\vec{k}_{\perp},\downarrow,\uparrow)|\downarrow\uparrow> \\ & + \psi(x,\vec{k}_{\perp},\uparrow,\uparrow)|\uparrow\uparrow> + \psi(x,\vec{k}_{\perp},\downarrow,\downarrow)|\downarrow\downarrow>, \end{aligned} \tag{2.1}$$

where

$$\psi(x,\vec{k}_\perp,\lambda_1,\lambda_2)=C_0^F(x,\vec{k}_\perp,\lambda_1,\lambda_2)\varphi(x,\vec{k}_\perp). \tag{2.2}$$

Here $\varphi(x,\vec{k}_\perp)$ is the light-cone momentum space wave function and $C_0^F(x,\vec{k}_\perp,\lambda_1,\lambda_2)$ represents the light-cone spin component coefficients for the total spin state $J=0$. When expressed in terms of the equal-time momentum $q^\mu=(q^0,\vec{q})$, the spin component coefficients have the forms,

$$\begin{array}{c}
C_0^F(x,\vec{k}_\perp,\uparrow,\downarrow)=w_1w_2[(q_1^+ +m)(q_2^+ +m)-\vec{q}_\perp^{\,2}]/\sqrt{2};\\
C_0^F(x,\vec{k}_\perp,\downarrow,\uparrow)=-w_1w_2[(q_1^+ +m)(q_2^+ +m)-\vec{q}_\perp^{\,2}]/\sqrt{2};\\
C_0^F(x,\vec{k}_\perp,\uparrow,\uparrow)=w_1w_2[(q_1^+ +m)q_2^L-(q_2^+ +m)q_1^L]/\sqrt{2};\\
C_0^F(x,\vec{k}_\perp,\downarrow,\downarrow)=w_1w_2[(q_1^+ +m)q_2^R-(q_2^+ +m)q_1^R]/\sqrt{2},
\end{array} \tag{2.3}$$

where $w=[2q^+(q^0+m)]^{-1/2}$, $q^{R,L}=q^1\pm i\ q^2$, and $q^+=q^0+q^3$. The relation between the equal-time momentum $\vec{q}=(q^3,\vec{q}_\perp)$ and the light-cone momentum $\underline{k}=(x,\vec{k}_\perp)$ [3, 24, 25] in this paper is:

$$\begin{array}{c}
xM\leftrightarrow(q^0+q^3);\\
\vec{k}_\perp\leftrightarrow\vec{q}_\perp,
\end{array} \tag{2.4}$$

in which M is defined as

$$M^2=\frac{\vec{k}_\perp^{\,2}+m^2}{x(1-x)} \tag{2.4 '}$$

From (2.4) it follows that

$$\frac{\vec{k}_\perp^{\,2}+m^2}{4x(1-x)}-m^2=\vec{q}^{\,2}. \tag{2.5}$$

We point out that the light-cone wave function (2.1) is the correct pion spin wave function since it is an eigenstate of the total spin operator $(\hat{S}^F)^2$ in the light-cone formalism. The equal-time and light-cone spin operators are related by the relation

$$\hat{S}^F=U\hat{S}^TU^{-1} \tag{2.6}$$

where U is the Wigner rotation operator. In the pion rest frame the spin of the pion is the vector sum of the equal-time spin of the two quarks in the case of zero orbital angular momentum,

$$\hat{S}^T=\hat{s}_1^T+\hat{s}_2^T. \tag{2.7}$$

We thus have the following relation

$$\hat{S}^F=\hat{S}^T=\hat{s}_1^T+\hat{s}_2^T=u_1^{-1}u_1\hat{s}_1^Tu_1^{-1}u_1+u_2^{-1}u_2\hat{s}_2^Tu_2^{-1}u_2=u_1^{-1}\hat{s}_1^Fu_1+u_2^{-1}\hat{s}_2^Fu_2 \tag{2.8}$$

taking into account the fact that it is invariant under the Wigner rotation for the spin-zero operator. This implies that the light-cone spin of a composite particle is not directly the sum of its constituents' light-cone spin but the sum of the Melosh rotated light-cone spin of the individual constituents. A natural consequence is that in light-cone formalism a hadron's helicity is not necessarily equal to the sum of the quark's helicities, i.e., $\lambda_H \neq \sum_i \lambda_i$. This result is important for understanding the proton "spin puzzle" [23].

From the helicity selection rules of Brodsky and Lepage [26] we know that there are no spin flip processes in perturbative interaction between quarks, thus the valence state equation for the pion can be obtained,

$$(M_\pi^2 - \frac{\vec{k}_\perp^2 + m^2}{x(1-x)})\psi(x, \vec{k}_\perp, \lambda_1, \lambda_2) = \int_0^1 dy \int_0^\infty \frac{d^2\vec{l}_\perp}{16\pi^3} < \lambda_1, \lambda_2 | V_{eff} | \lambda_1, \lambda_2 > \psi(y, \vec{l}_\perp, \lambda_1, \lambda_2). \tag{2.9}$$

Some effects of all higher Fock states are included in V_{eff} and the valence state plays a special role in the high-momentum form factor, so the above equation will be useful in our pQCD analysis in the following section.

III. CONTRIBUTIONS FROM THE HIGHER HELICITY COMPONENTS

In the conventional equal-time formalism, there is a Wigner rotation between spin states in different frames and the consequences of the Wigner rotation should be carefully taken into account. The spin structure of a composite system will be different in different frames, and this aspect of the composite spin should be considered. It may be reasonable to neglect this effect in some cases when the relativistic effects of the Wigner rotation are small. However, in higher momentum transfer processes these effects should not be ignored because the relativistic effects become large. One advantage of the light-cone formalism is that the Wigner rotation relating spin states in different frames is unity under a kinematic Lorentz transformation; thereby the spin structure of hadrons is the same in different frames related by a kinematic Lorentz transformation [27]. However, the spin structure of a composite system is now much different from that in the equal-time formalism in the rest frame of the composite system if the relativistic effects are considered. So the consequences of the Wigner rotation are contained in the contributions from higher helicity components in the light-cone formalism.

An exact expression for the pion's electromagnetic form factor is the Drell-Yan-West formula [28]

$$F(Q^2) = \sum_{n,\lambda_i} \sum_j e_j \int [dx][d^2\vec{k}_\perp] \psi_n^*(x_i, \vec{k}_{\perp i}, \lambda_i) \psi_n(x_i, \vec{k}'_{\perp i}, \lambda_i), \tag{3.1}$$

where $\vec{k}'_\perp = \vec{k}_\perp - x_i \vec{q}_\perp + \vec{q}_\perp$ for the struck quark, $\vec{k}'_\perp = \vec{k}_{\perp i} - x_i \vec{q}_\perp$ for the spectator quarks and e_i is the quark's electric charge. For high momentum transfer (3.1) can be approximated in terms of the q$\bar{\mathrm{q}}$ wave function which dominates over all other Fock states

$$F(Q^2) = \sum_{\lambda_1,\lambda_2} \int \frac{dx\ d^2\vec{k}_\perp}{16\pi^3} \int \frac{dy\ d^2\vec{l}_\perp}{16\pi^3} \psi^*(x, \vec{k}_\perp, \lambda_1, \lambda_2)$$
$$\frac{T(x, \vec{k}_\perp; y, \vec{l}_\perp; \vec{q}_\perp)}{[x(1-x)y(1-y)]^{1/2}} \psi(y, \vec{l}_\perp, \lambda_1, \lambda_2), \tag{3.2}$$

where T is the sum of all q$\bar{\mathrm{q}}$ irreducible light-cone perturbative amplitudes contributing to the q$\bar{\mathrm{q}}$ form factor γ^*+q$\bar{\mathrm{q}}\rightarrow$ q$\bar{\mathrm{q}}$ [1-3,9]. Considering first the disconnected part of T, and ignoring the renormalization diagrams and taking into account only terms where the photon attaches to the quark line, we have then the disconnected part contributions

$$F(Q^2) = \sum_{\lambda_1,\lambda_2} \sum_j e_j \int \frac{dx\ d^2\vec{k}_\perp}{16\pi^3} \psi^*(x, \vec{k}_\perp, \lambda_1, \lambda_2) \psi(x, \vec{l}_\perp, \lambda_1, \lambda_2). \tag{3.3}$$

In conventional pQCD analyses [1-3,9] only the ordinary helicity ($\lambda_1 + \lambda_2 = 0$) components were considered and it was argued that Eq.(3.3) could be replaced by the factorized formula

$$F(Q^2) = \int_0^1 dx \int_0^1 dy \phi^{0\,*}(x, (1-x)Q) T_H(x, y; Q) \phi^0(y, (1-y)Q), \tag{3.4}$$

where

$$T_H = 16\pi C_F (e_u \frac{\alpha_s[(1-x)(1-y)Q^2]}{(1-x)(1-y)Q^2} + e_{\overline{d}} \frac{\alpha_s(xyQ^2)}{xyQ^2}) \tag{3.5}$$

is the leading hard scattering amplitude for scattering collinear constituents q and $\bar{\mathrm{q}}$ from the initial to final direction, and

$$\phi^0(x, Q) = \int^{Q^2} \frac{d^2\vec{k}_\perp}{16\pi^3} \psi^0(x, \vec{k}_\perp) \tag{3.6}$$

is the distribution amplitude for the $\lambda_1 + \lambda_2 = 0$ components. In the above formulas, $e_u = 2/3$, $e_{\overline{d}} = 1/3$ are the quark charges, $C_F = 4/3$ is the value of the

Casimir operator for the fundamental representation of SU(3) (i.e., the quark's representation), and $\alpha_s(Q^2) = 4\pi/\beta_0 ln(Q^2/\Lambda^2_{QCD})$ is the running coupling constant of QCD with scale parameter $\Lambda_{QCD} \approx 200$MeV and $\beta_0 = 11 - 2n_f/3$ where n_f is the number of active quark flavours. If the factorized formula (3.5) is also applicable to the $\lambda_1 + \lambda_2 = \pm 1$ components, then one will naturally arrive at the conclusion that the perturbative contributions from the $\lambda_1 + \lambda_2 = \pm 1$ components are negligible because the distribution amplitudes for these components vanish [8] and that neglecting $\lambda_1 + \lambda_2 = \pm 1$ components in conventional pQCD analyses is thus a reasonable approximation.

The contributions from the ordinary and higher helicity components, when calculated from (3.3), should be analysed separately. First, in the case of $\lambda_1 + \lambda_2 = 0$ components, we have

$$F^0_\pi(Q^2) = \int \frac{dx\, d^2\vec{k}_\perp}{16\pi^3} \psi^{0\,*}(x, \vec{k}_\perp)\psi^0(x, \vec{k}'_\perp), \tag{3.7}$$

where

$$\psi^0(x, \vec{k}_\perp) = \sqrt{2}\psi(x, \vec{k}_\perp, \lambda_1, \lambda_2) = \frac{a_1 a_2 - \vec{k}^2_\perp}{[(a^2_1 + \vec{k}^2_\perp)(a^2_2 + \vec{k}^2_\perp)]^{1/2}} \varphi(x, \vec{k}_\perp), \tag{3.8}$$

with $a_i = x_i M + m$ and $a'_i = x_i M' + m$, is the ordinary helicity wave function and is assumed to satisfy the equation, for $< \vec{k}^2_\perp > \ll \vec{q}^2_\perp$,

$$\psi^0(x, (1-x)\vec{q}_\perp) \approx \int_0^1 dy \frac{V_{eff}(x, (1-x)\vec{q}_\perp; y, \vec{0}_\perp)}{-\vec{q}^2_\perp(1-x)/x} \phi^0(y, (1-y)Q), \tag{3.9}$$

where V_{eff} is given by $V_{eff} = T_H/[x(1-x)y(1-y)]^{1/2}$. The arguments in deriving the factorized formula in conventional pQCD analyses [1-3,9] are applicable to (3.7). The integral is dominated by two regions of phase space when Q^2 is large since the wave functions, $\psi^0(x, \vec{k}_\perp)$ and $\psi^0(x, \vec{k}'_\perp)$ are sharply peaked at low transverse momentum:

(1). $\vec{k}^2_\perp \ll Q^2$ where $\psi^{0\,*}(x, \vec{k}_\perp)$ is large;

(2). $\vec{k}'^2_\perp = (\vec{k}_\perp + (1-x)\vec{q}_\perp)^2 \ll Q^2$ where $\psi^0(x, \vec{k}_\perp + (1-x)\vec{q}_\perp)$ is large.

In region (1), $\vec{k}_\perp$ can be neglected in $\psi^0(x, \vec{k}_\perp + (1-x)\vec{q}_\perp)$ until $|\vec{k}_\perp| \approx (1-x)Q$, at which point ψ^0 begins to cut off the $\vec{k}_\perp$ integration. Thus in region (1) we can approximate (3.7) by

$$F^0_{(1)} = \int_0^1 dx \psi^0(x, (1-x)\vec{q}_\perp) \int^{(1-x)Q} \frac{d^2\vec{k}_\perp}{16\pi^3} \psi^{0\,*}(x, \vec{k}_\perp) \tag{3.10}$$

From (3.9) we have,

$$F^0_{(1)} = \int_0^1 dx \int_0^1 dy \phi^0(y,(1-y)Q) \frac{V_{eff}(x,(1-x)\vec{q}_\perp; y, \vec{0}_\perp)}{-\vec{q}_\perp^{\,2}(1-x)/x} \phi^{0\,*}(x,(1-x)Q). \tag{3.11}$$

Similarly we can approximate the contribution to (3.7) for region (2)

$$F^0_{(2)} = \int_0^1 dx \int_0^1 dy \phi^{0\,*}(y,(1-y)Q) \frac{V_{eff}(x,(1-x)\vec{q}_\perp; y, \vec{0}_\perp)}{-\vec{q}_\perp^{\,2}(1-x)/x} \phi^{0}(x,(1-x)Q). \tag{3.12}$$

Therefore we arrive at the factorized formula

$$F^0_\pi = \int_0^1 dx \int_0^1 dy \phi^{0\,*}(x,(1-x)Q) T_H(x,y;Q) \phi^0(y,(1-y)Q). \tag{3.13}$$

However, the situation is different in the case of $\lambda_1+\lambda_2=\pm 1$. The contributions to the pion form factor from these components, $F^{\pm 1}(Q^2)$, can be written as

$$\begin{aligned} F^{\pm 1}_\pi(Q^2) &= \int \frac{dx\, d^2\vec{k}_\perp}{16\pi^3} \frac{(a_1+a_2)(a'_1+a'_2)\vec{k}_\perp\cdot\vec{k}'_\perp}{[(a_1^2+\vec{k}_\perp^2)(a_2^2+\vec{k}_\perp^2)(a_1'^2+\vec{k}_\perp'^2)(a_2'^2+\vec{k}_\perp'^2)]^{1/2}} \\ &\qquad \varphi^*(x,\vec{k}_\perp)\varphi(x,\vec{k}'_\perp) \\ &= \int \frac{dx\, d^2\vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp\cdot\vec{k}'_\perp}{m^2} \psi^{0\,*}(x,\vec{k}_\perp)\psi^0(x,\vec{k}'_\perp). \end{aligned} \tag{3.14}$$

Although $\int d^2\vec{k}_\perp \vec{k}_\perp \psi^0(x,\vec{k}_\perp)=0$, we can not ignore their contributions since there is a factor $\vec{k}_\perp\cdot\vec{k}'_\perp = \vec{k}_\perp^2+(1-x)\vec{k}_\perp\cdot\vec{q}_\perp$ which may cause non-vanishing contributions. In order to evaluate the contributions from the $\lambda_1+\lambda_2=\pm 1$ components, we use the identity $\vec{k}_\perp\cdot\vec{k}'_\perp = 1/2\,[\vec{k}_\perp^2+\vec{k}_\perp'^2-(1-x)^2\vec{q}_\perp^{\,2}]$ and re-express (3.14) as

$$\begin{aligned} F^{\pm 1}_\pi(Q^2) = \int \frac{dx\, d^2\vec{k}_\perp}{16\pi^3} [&\underbrace{\psi^{0\,*}(x,\vec{k}_\perp)\tilde{\psi}(x,\vec{k}'_\perp)}_{(a)} + \underbrace{\tilde{\psi}^*(x,\vec{k}_\perp)\psi^0(x,\vec{k}'_\perp)}_{(b)} \\ &- \underbrace{\frac{(1-x)^2\vec{q}_\perp^{\,2}}{2m^2}\psi^{0\,*}(x,\vec{k}_\perp)\psi^0(x,\vec{k}'_\perp)}_{(c)}], \end{aligned} \tag{3.15}$$

where

$$\tilde{\psi}(x,\vec{k}_\perp) = \frac{\vec{k}_\perp^2}{2m^2}\psi^0(x,\vec{k}_\perp). \tag{3.16}$$

It may be seen that the factorization arguments can be applied to the three terms of (3.15) respectively if the "new" wave function $\tilde{\psi}(x,\vec{k}_\perp)$ is also sharply peaked at low

transverse momentum. By arguments similar to the above, one can approximate the contribution to term (a) for region (1),

$$F^{\pm 1}_{(a)(1)} = \int_0^1 dx \tilde{\psi}(x, (1-x)\vec{q}_\perp) \int^{(1-x)Q} \frac{d^2\vec{k}_\perp}{16\pi^3} \psi^{0\,*}(x, \vec{k}_\perp). \tag{3.17}$$

From (3.16) and (3.9), we know, for $< \vec{k}_\perp^2 > \ll \vec{q}_\perp^2$,

$$\tilde{\psi}(x, (1-x)\vec{q}_\perp) = \frac{(1-x)^2\vec{q}_\perp^2}{2m^2} \psi^0(x, (1-x)\vec{q}_\perp)$$
$$\approx \frac{(1-x)^2\vec{q}_\perp^2}{2m^2} \int_0^1 dy \frac{V_{eff}(x, (1-x)\vec{q}_\perp; y, \vec{0}_\perp)}{-\vec{q}_\perp^2(1-x)/x} \phi^0(y, (1-y)Q). \tag{3.18}$$

Thus we have

$$F^{\pm 1}_{(a)(1)} = \int_0^1 dx \int_0^1 dy \frac{(1-x)^2\vec{q}_\perp^2}{2m^2}$$
$$\phi^0(y, (1-y)Q) \frac{V_{eff}(x, (1-x)\vec{q}_\perp; y, \vec{0}_\perp)}{-\vec{q}_\perp^2(1-x)/x} \phi^{0\,*}(x, (1-x)Q). \tag{3.19}$$

The contribution to term (a) for region (2) can be approximated by

$$F^{\pm 1}_{(a)(2)} = \int_0^1 dx \psi^{0\,*}(x, (1-x)\vec{q}_\perp) \int^{(1-x)Q} \frac{d^2\vec{k}_\perp}{16\pi^3} \tilde{\psi}(x, \vec{k}_\perp). \tag{3.20}$$

Thus $F^{\pm 1}_{(a)(2)}$ becomes

$$F^{\pm 1}_{(a)(2)} = \int_0^1 dx \int_0^1 dy \tilde{\phi}(y, (1-y)Q) \frac{V_{eff}(x, (1-x)\vec{q}_\perp; y, \vec{0}_\perp)}{-\vec{q}_\perp^2(1-x)/x} \phi^{0\,*}(x, (1-x)Q), \tag{3.21}$$

where

$$\tilde{\phi}(x, Q) = \int^{Q^2} \frac{d^2\vec{k}_\perp}{16\pi^3} \tilde{\psi}(x, \vec{k}_\perp). \tag{3.22}$$

In analogy to the above arguments, we have

$$F^{\pm 1}_{(b)(1)} = \int_0^1 dx \int_0^1 dy \phi^0(y, (1-y)Q) \frac{V_{eff}(x, (1-x)\vec{q}_\perp; y, \vec{0}_\perp)}{-\vec{q}_\perp^2(1-x)/x} \tilde{\phi}^*(x, (1-x)Q), \tag{3.23}$$

$$F^{\pm 1}_{(b)(2)} = \int_0^1 dx \int_0^1 dy \frac{(1-x)^2\vec{q}_\perp^2}{2m^2}$$
$$\phi^{0\,*}(y, (1-y)Q) \frac{V_{eff}(x, (1-x)\vec{q}_\perp; y, \vec{0}_\perp)}{-\vec{q}_\perp^2(1-x)/x} \phi^0(x, (1-x)Q), \tag{3.24}$$

$$F^{\pm 1}_{(c)(1)} = \int_0^1 dx \int_0^1 dy \frac{(1-x)^2 \vec{q}_\perp^2}{2m^2}$$
$$\phi^{0\,*}(y,(1-y)Q) \frac{V_{eff}(x,(1-x)\vec{q}_\perp; y, \vec{0}_\perp)}{-\vec{q}_\perp^2 (1-x)/x} \phi^0(x,(1-x)Q), \qquad (3.25)$$

$$F^{\pm 1}_{(c)(2)} = \int_0^1 dx \int_0^1 dy \frac{(1-x)^2 \vec{q}_\perp^2}{2m^2}$$
$$\phi^{0}(y,(1-y)Q) \frac{V_{eff}(x,(1-x)\vec{q}_\perp; y, \vec{0}_\perp)}{-\vec{q}_\perp^2 (1-x)/x} \phi^{0\,*}(x,(1-x)Q). \qquad (3.26)$$

Combining (3.19), (3.21), (3.23)-(3.26) together, we have, for (3.15),

$$F^{\pm 1}_\pi = \int_0^1 dx \int_0^1 dy \phi^{0\,*}(x,(1-x)Q) T_H(x,y;Q) \tilde{\phi}(y,(1-y)Q), \qquad (3.27)$$

which is essentially a new factorized formula for the perturbative contributions from the higher helicity components.

We point out that the factorized formula (3.27) could be as reliable as (3.4) if $\tilde{\psi}(x,\vec{k}_\perp)$ is sharply peaked at low transverse momentum. However, $\tilde{\psi}(x,\vec{k}_\perp)$ seems to fall off exponentially at large transverse momentum, as seen from (3.18). We thus can only consider (3.27) as an evaluation of the perturbative contributions to $F^{\pm 1}_\pi$ within regions (1) and (2). There are possible non-perturbative contributions outside the two regions (1) and (2). Nevertheless, terms (a) and (b) in (3.15) should have positive contributions to $F^{\pm 1}_\pi$ whereas term (c) seems can be negligible because the dominant contributions are from regions (1) and (2) as argued in deriving (3.13). Thus we can consider (3.27) as a first crude evaluation of the perturbative contributions from the higher helicity components before an improved result is obtained.

IV. NUMERICAL RESULTS AND TEST OF THE APPROACH

In order to calculate perturbative contributions to the pion form factor, we need to know, besides the ordinary distribution amplitude $\phi^0(x,Q)$, the explicit form of the new distribution amplitude $\tilde{\phi}(x,Q)$. From Sec. I we know that even the approximate form of the ordinary distribution amplitude is still an open problem. In this paper we adopt the revised light-cone quark model approach [17] to evaluate the ordinary and the new distribution amplitudes.

A recent lattice calculation [16] gives a small value of the second moment for the distribution amplitude $\phi^0(x,Q)$. This suggests that the ordinary pion distribution amplitude may be close to the asymptotic form rather than the CZ form. Therefore it is suitable to adopt the Brodsky-Huang-Lepage prescription [3] for the momentum space wave function in the light-cone formalism

$$\varphi(x,\vec{k}_\perp) = A\ exp[-\frac{m^2+\vec{k}_\perp^2}{8\beta^2 x(1-x)}], \tag{4.1}$$

with the parameters adjusted by fitting several physical constraints [17] to approximate $\phi^0(x)$ and $\tilde{\phi}(x)$ at moderate values of Q^2. Because the QCD evolution of the distribution amplitudes is very slow [3], we will neglect the Q^2 dependence of $\phi^0(x)$ and $\tilde{\phi}(x)$ in our numerical calculations. The calculated second moments for $\phi^0(x)$ and $\tilde{\phi}(x)$ are 0.191 and 0.221 respectively, which are close to that of the asymptotic form (0.2) rather than that of the CZ form (0.4). Figure 1 presents the calculated distribution amplitudes $\phi^0(x)$ and $\tilde{\phi}(x)$. We see that both of them are close to the asymptotic form, with $\tilde{\phi}(x)$ more narrow than $\phi^0(x)$ and thus its magnitude is higher than that of $\phi^0(x)$ in the middle x region. The two distribution amplitudes are highly suppressed in the end-point regions, thus the perturbative results obtained by using these distribution amplitudes do not suffer from the first objection of pQCD by Isgur and Llewellyn Smith [8]. However, the second objection by Isgur and Llewellyn Smith still remains because there are possible non-perturbative contributions to $F_\pi^{\pm 1}$ outside regions (1) and (2). Figure 2 presents the calculated perturbative contributions to the pion form factor. We see that the contributions from the ordinary helicity components and the higher helicity components each account for almost half of the existing pion form factor data [29]. The sum of them could give a good description of the data at currently available momentum transfers even down to Q^2 of about 1-2(GeV/c)^2. Of course, the use of other wave functions will alter our quantitative results and we need to use the improved pQCD approach [11, 21] if the distribution amplitudes are not negligible in the end-point regions. Progress has been made in this direction and will be given elsewhere.

We point out that whether the higher helicity components contribute to the pion form factor is essentially a result that can be tested from comparison with other exclusive processes. Though the higher helicity components contribute to the pion form factor, they do not contribute to the π-γ transition form factor $F_{\pi\gamma}$ discussed by Lepage and Brodsky [2], since the helicity selection rules [26] require the quark's helicity to be conserved at the qγ $\rightarrow$q vertexes in figure 2 of [2]. The

perturbative contribution to $F_{\pi\gamma}$ can be written as

$$F_{\pi\gamma}(Q^2) = \int_0^1 dx_1 dx_2 \delta(1 - x_1 - x_2) T'_H(x_i, Q) \phi(x_i, \tilde{Q}) \tag{4.2}$$

(i.e., equation (2.11) in [2]), where T'_H is the hard-scattering amplitude for γ^*+q$\bar{\text{q}}$$\rightarrow$ γ with on-shell collinear quarks (i.e., equation (2.12) in [2])

$$T'_H = \frac{2\sqrt{n_c}(e_u^2 - e_d^2)}{x_1 x_2 Q^2}. \tag{4.3}$$

If the higher helicity components do not contribute to the pion form factor, then one can predict, at large Q^2,

$$\alpha(Q^2) \approx \frac{n_c(e_u^2 - e_d^2)^2}{\pi C_F} \frac{F_\pi(Q^2)}{Q^2 |F_{\pi\gamma}(Q^2)|^2} + O(\alpha_s^2(Q^2)) \approx \frac{1}{4\pi} \frac{F_\pi(Q^2)}{Q^2 |F_{\pi\gamma}(Q^2)|^2}. \tag{4.4}$$

which is equation (4.6) in [2]; whereas one should have

$$\alpha(Q^2) < \frac{1}{4\pi} \frac{F_\pi(Q^2)}{Q^2 |F_{\pi\gamma}(Q^2)|^2}. \tag{4.5}$$

in the case that the higher helicity components contribute to the pion form factor. The value of the right side of (4.5) can be two times of the left side if the higher helicity contribution to the pion form factor is of the same magnitude as the ordinary helicity contribution, as evaluated above. It is not difficult to judge experimentally whether there are higher helicity contributions to the pion form factor. Thus we can test the approach of this paper by measuring both the pion form factor F_π and the π-γ transition form factor $F_{\pi\gamma}$ at large Q^2. Of course, consideration of further possible effects due to the axial-anomaly and contributions from multiparticle wave functions, higher twists[30], and soft mechanism[18] etc. could complicate the above analysis.

We need to address the difference between the results in this paper and those in [19]. Ref. [19] is a constituent quark model evaluation of the Wigner rotation effect on the low energy properties of the pion. The result of the pion form factor is only valid at low Q^2, from $Q^2 = 0$ to 1 (GeV/c)^2, as indicated in [19]. Whereas the present work is to discuss the pion form factor at large Q^2, where the perturbative analysis is applicable. In principle the perturbative analysis should be valid at $Q^2 \rightarrow \infty$. However, it is still an open problem at which value of Q^2 the perturbative results are still valid, as have been extensive discussed in a number of literature, e.g.[8-11]. In our paper the applicability of perturbative results seem to be extended

to Q^2 of several (GeV/c)^2, at which the constituent quark model results begin to fail. The constituent quark model analysis of the pion form factor is not applicable to large Q^2, since the calculated result at large Q^2 is very sensitive to the explicit high momentum tail behaviour of the pion wave function and at present there is no reliable non-perturbative result of the pion wave function. The perturbative analysis in this paper is not applicable to low Q^2, since (3.9) is only valid at large Q^2. Thus the results of [19] and the present paper are applicable to different Q^2 regions.

V. SUMMARY AND DISCUSSION

As is well known, there has been many significant progress in applying perturbative QCD to exclusive processes. However, its applicability is still under debate because of the ambiguous understanding of the structure of hadrons. For example, one still can not precisely determine the process-independent "distribution amplitudes" which are related to the full light-cone wave function. In this paper we reanalyse the perturbative calculation of the pion form factor and find that the Wigner rotation plays a role in the application of perturbative QCD to exclusive processes.

We should make it clear that the Wigner rotation is treated in the paper without considering any dynamical effect due to quark interactions in the boost from the pion rest reference frame to the infinite momentum frame for the spin part wave function. The inclusion of dynamical effects may change the explicit expressions and quantitative results in the paper. However, the kinematics should be considered first before the introduction of other dynamical effects. Therefore one has to evaluate the contributions due to the Wigner rotation in the application of perturbative QCD to exclusive processes.

In many previous pQCD analyses of exclusive processes the consequence of the Wigner rotation was not considered. The results in this paper suggest that we should carefully analyse the consequence of the Wigner rotation in a specific process, rather than apply the conventional factorization formula to the higher helicity components. The introduction of the higher helicity components into perturbative QCD theory may shed some light on several other problems concerning the applicability of perturbative QCD in the high momentum transfer region. The higher helicity components are likely the sources for the "helicity non-conserving" behaviours [31] observed in pp$^\uparrow$ scattering [32] and in the process πN$\rightarrow$ ρN [33]. In the sense of this paper the "helicity non-conserving" behaviours do not really mean

helicity non-conserving as recognized previously but the necessary of considering the higher helicity components in the initial and final hadrons. Thus our speculation is not in conflict with the helicity selection rules [26]. The only modification is that the hadron's helicity should not equal the sum of the quark's helicities, as argued in Sec. II.

As an example, we re-examined in this paper the pion form factor and derived the contribution from the higher helicity components. This contribution may provide part of the other fraction needed to fit the pion form factor data besides the perturbative contributions from ordinary helicity components. However, it is also possible that this contribution may have a Q^2 suppressed behavior and vanish at very large Q^2. Then we will return to the conventional results at large Q^2. A way to test this contribution has been suggested. This result is consistent quantitatively with the dimensional counting rule, and the applicability of perturbative QCD to exclusive processes seems to be extended to momentum transfer of several $(\mathrm{GeV/c})^2$.

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Note after publication The quantitative conclusion of this paper may change from our recent investigation.

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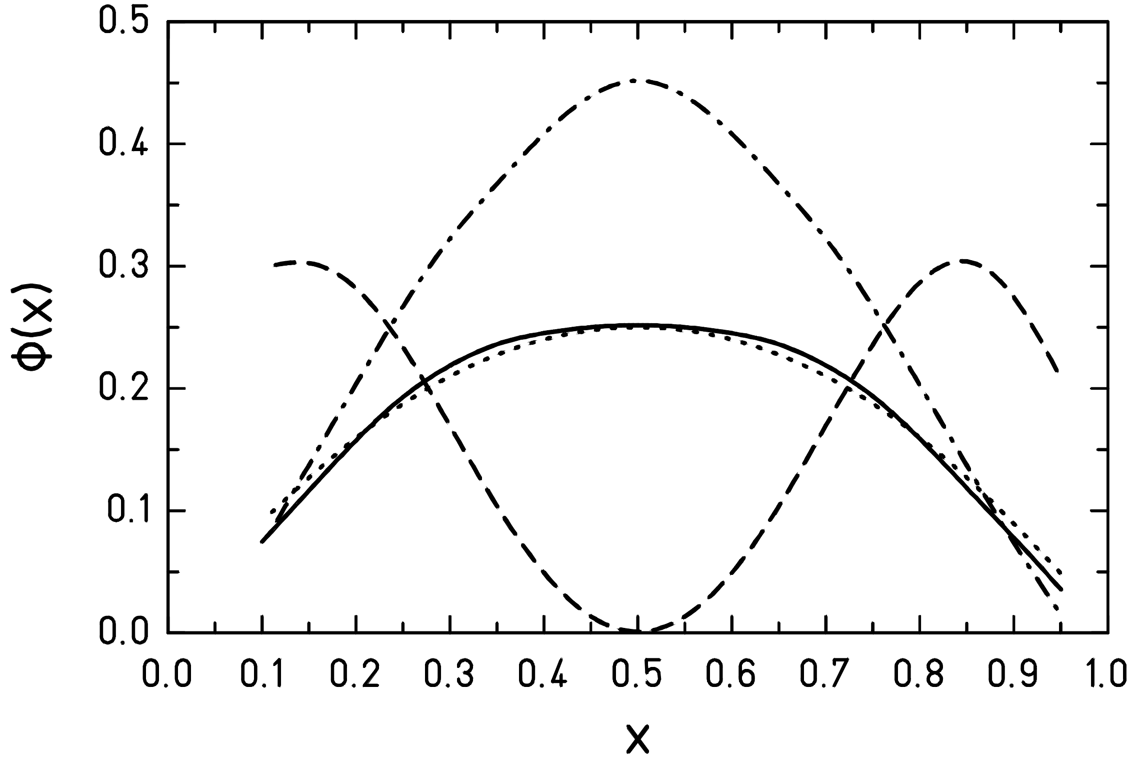
Figure Captions

Fig. 1. The normalized distribution amplitude $\hat{\phi}(x) = \phi(x)/\sqrt{3}f_\pi$: the dotted and dashed curves are the asymptotic distribution amplitude [3] and the Chernyak-Zhitnitsky distribution amplitude [12]; the solid and dot-dashed curves are the distribution amplitudes $\phi^0(x)$ and $\tilde{\phi}(x)$ evaluated in the revised light-cone quark model approach [17] with parameters $m = 330$MeV and $\beta = 540$MeV adjusted by fitting several physical constraints.

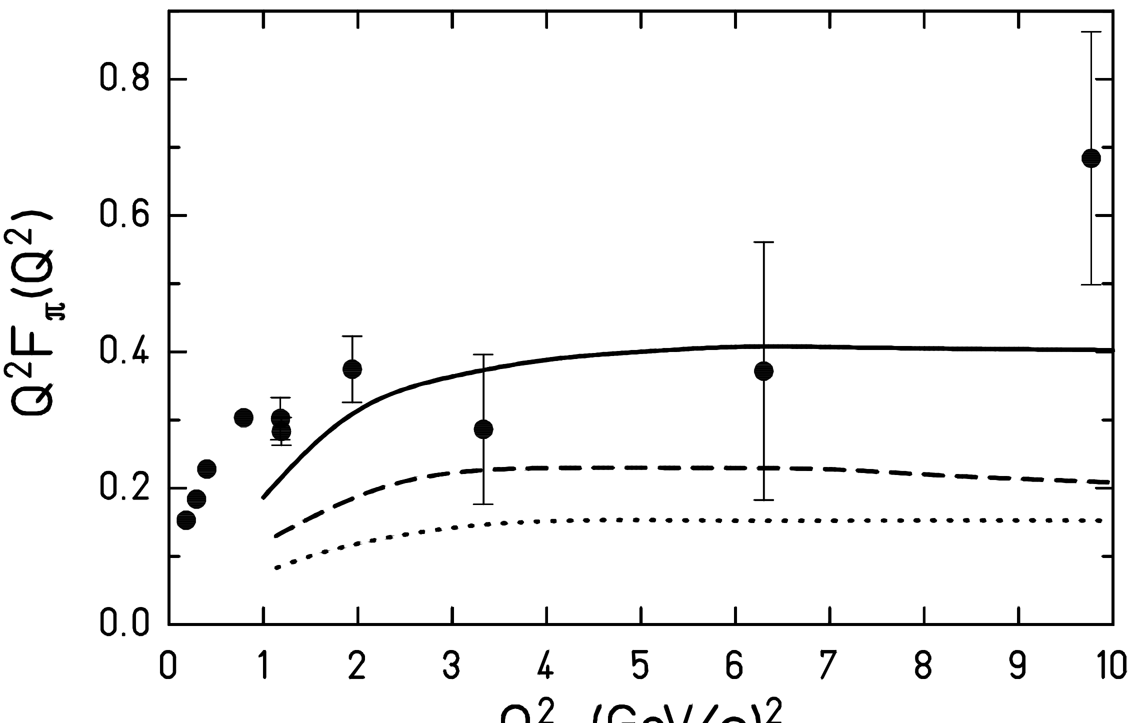
Fig. 2. The perturbative contributions to the pion form factors calculated using the factorized formulas (3.4) and (3.27). The dotted and dashed curves are the contributions from the ordinary and higher helicity components respectively, and the solid curve is the sum of them with the QCD scale parameter $\Lambda_{QCD} = 200$MeV. The data are from [29].

Fig.1



'UR59.PWF2FIG1.PLOT'

Fig.2



'UR59.PWF2FIG2.PLOT'